group Γ. Its coefficients possess many remarkable arithmetic properties, which are set forth in the appended references. For example, the congruence

$$(n + 1)c(n) \equiv 0 \pmod{24},$$

due to D. H. Lehmer [2], implies that $c(n)$ is even except possibly when $n \equiv 7 \pmod 8$. In this case it may be shown that $c(n)$ assumes both even and odd values infinitely often, although necessary and sufficient conditions for $c(n)$ to be odd are still unknown.

The coefficients were first computed for $-1 \leqslant n \leqslant 24$ by H. S. Zuckerman [7] and then for $-1 \leqslant n \leqslant 100$ by A. van Wijngaarden [6]. Here we tabulate the coefficients for $-1 \leqslant n \leqslant 500$. There would seem to be little point in extending the table further, since $c(500)$ is already a number of 120 digits.

The coefficients were computed, using residue arithmetic, by means of the following formula [5]:

$$c(n) = p_{-24}(n + 1) + \frac{65520}{691} \sum_{k=0}^{n} \sigma_{11}(k + 1)p_{-24}(n - k), \qquad n \geqslant 1,$$

where $\Sigma_{n=0}^{\infty} p_{-24}(n)x^n = \Pi_{n=1}^{\infty} (1 - x^n)^{-24}$.

The total computation time on a UNIVAC 1108 system was approximately four minutes.

AUTHOR'S SUMMARY

1. O. KOLBERG, "Congruences for the coefficients of the modular invariant $j(\tau)$ modulo powers of 2," *Univ. Bergen Arbok Naturvit. Rekke*, v. 16, 1961.

2. D. H. LEHMER, "Properties of the coefficients of the modular invariant $J(\tau)$," *Amer. J. Math.*, v. 64, 1942, pp. 488–502.

3. J. LEHNER, "Divisibility properties of the Fourier coefficients of the modular invariant $J(\tau)$," *Amer. J. Math.*, v. 71, 1949, pp. 136–148.

4. J. LEHNER, "Further congruence properties of the Fourier coefficients of the modular invariant $J(\tau)$," *Amer. J. Math.*, v. 71, 1949, pp. 337–386.

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6. A. VAN WIJNGAARDEN, "On the coefficients of the modular invariant $J(\tau)$," *Nederl. Akad. Wetensch. Proc. Ser. A*, v. 16, 1953, pp. 389–400.

7. H. S. ZUCKERMAN, "The computation of the smaller coefficients of $J(\tau)$," *Bull. Amer. Math. Soc.*, v. 45, 1939, pp. 917–919.

12 [9].—DANIEL SHANKS, *Table of the Greatest Prime Factor of* $N^2 + 1$ *for* $N = 1(1)185000$, 1959, two ms. volumes, each of 185 computer sheets, bound in cardboard covers and deposited in the UMT file.

This table was calculated in 1959 on an IBM 704 system by the p-adic sieve method described completely in [1]. The method is extraordinarily efficient: each division performed is known a priori to have a zero remainder. From the complete factorization of $n^2 + 1$ for $n = 1(1)185000$ I then tabulated only the greatest prime factors, 500 per page, arranged in an obvious format. (One can see at once which $n^2 + 1$ are prime by the relative size of the corresponding listed factors.)

These factorizations relate to questions concerning reducible numbers, primes of the form $n^2 + 1$, formulas for π, and other questions surveyed in [1].

In [2] and [3] similar p-adic sieves were run for $n^2 \pm 2$, $n^2 \pm 3$, $n^2 + 4$, $n^2 \pm 5$, $n^2 \pm 6$, and $n^2 \pm 7$ for $n = 1(1)180000$, but only statistical information was preserved, not the complete table of greatest prime factors.

D. S.

1. DANIEL SHANKS, "A sieve method for factoring numbers of the form $n^2 + 1$," *MTAC*, v. 13, 1959, pp. 78–86.
2. DANIEL SHANKS, "On the conjecture of Hardy & Littlewood concerning the number of primes of the form $n^2 + a$," *Math. Comp.*, v. 14, 1960, pp. 321–332.
3. DANIEL SHANKS, "Supplementary data and remarks concerning a Hardy-Littlewood conjecture," *Math. Comp.*, v. 17, 1963, pp. 188–193.

13 [9].—J. D. SWIFT, *Table of Carmichael Numbers to* 10^9, University of California at Los Angeles, ms. of 20 pages, $8\frac{1}{2}'' \times 11''$, deposited in the UMT file.

A Carmichael number, CN, is a composite number n such that $a^{n-1} \equiv 1 \pmod{n}$ whenever $(a, n) = 1$. Carmichael numbers are starred in Poulet's table [1] of pseudoprimes less than 10^8. The present table corrects that table and extends the range to 10^9. The CN's are given with their prime factors.

Calculations were performed on a CDC 1604 made available by IDA, in Princeton. The computer programs used depended explicitly on congruence properties of CN's with respect to their component primes rather than on the pseudoprimality with respect to any particular base. A different routine was run for each possible number of primes occurring in the factorization, from 3 (the absolute minimum) to 6 (the effective maximum defined by the upper limit of the table).

For example, consider $n = p_1p_2p_3 = (r_1 + 1)(r_2 + 1)(r_3 + 1)$. The basic criteria are that $r_i \mid p_jp_k - 1$ where i, j, k is a permutation of 1, 2, 3. For a fixed choice of p_1 (assuming $p_1 < p_2 < p_3$), bounds on the limits of the calculation are obtained. In this simplest case an explicit bound is available:

$$p_1p_2p_3 \leqslant (p_1^6 + 2p_1^5 - p_1^4 - p_1^3 + 2p_1^2 - p_1)/2;$$

and this is actually a CN for $p_1 = 3, 5, 31, \cdots$ (?).

The total number of CN's less than or equal to each power of 10 is as follows:

x	CN(x)	ratio
10^4	7	
		2.3
10^5	16	
		2.7
10^6	43	
		2.4
10^7	105	
		2.4
10^8	255	
		2.5
10^9	646	